Grasshopper Tours

material from Václav Kotšovec

The Grasshopper moves along queen lines, but must jump one man (the “hurdle”) and land on the square immediately beyond (so a G on an empty board cannot move). The hurdle may be of either colour.

The idea of a grasshopper tour seems first to have been broached by S. H. Hall in Fairy Chess Review in February 1938. He used a knight, the knight and grasshopper moving alternately, but we’ll come back to this later because his solution wasn’t quite optimal and there has been progress since. It prompted Dawson to try the task using a rook (April 1938), which gave a very elegant solution:

The grasshopper starts at a7, the rook at a2, and we play Ga1, Rb2, Gc3, Re2, Gc1, Rb2, Ga3, and so on. At the end, with the grasshopper on a4, we play Ra3, Ga2, Ra6, Ga7, Ra2, and round we go again. I have renumbered to highlight the pattern (Dawson started from Ga1/Rb2). We start by visiting the dark squares on the odd files, starting at a7 and ending at g1. From there, we go to b1 and visit the light squares on the even files, and so on round. Numbers in diametrically opposite squares always differ by 32.

In Chessics 5 (July 1978), C. M. B. Tylor looked at the task with a king, and found a solution with a similar four-fold symmetry:

This time we work round the board quarter by quarter, starting with Gc4 and Kb4, visiting each square in the lower left quarter finishing at c1, then moving to e3 and visiting each square in the lower right quarter, and so on.

Now to the knight. Hall, working by hand, found a 60-move open tour:

Nobody else, not even Dawson, did better than 57. This remained the target until T. H. Willcocks produced a 63-move open tour (Chessics 23, Autumn 1985):

Additionally, the tour from I round to 62 is cyclic.

Willcocks, like Hall, appears to have worked by hand. Václav has now found a 64-move closed tour by computer:

Although closed, this is not cyclic (we end with the knight on b1, and cannot go round again). Václav tells me that the existence of a cyclic 64-move tour, although not yet proved impossible, appears unlikely.

All this, and much more, is in a book “Dual-free Leaper and Hopper Tours” recently produced by Václav in Praha (Prague). It is in Czech, but notes in English highlight what is going on, and the tables and diagrams are self-explanatory.